Addition

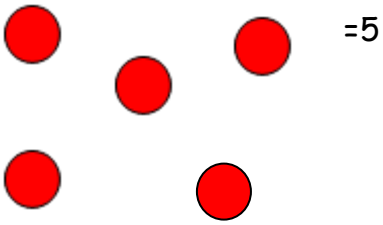
The term 'ones' will be used to replace 'units'.

HTO will replace HTU

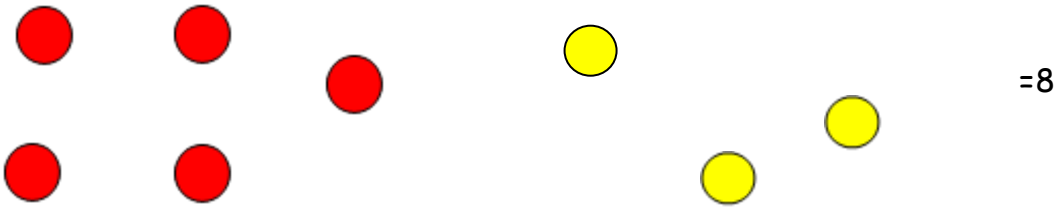
The majority of children should be using the refined column method before leaving Y6.

Stage 1 – Various practical activities to ensure a conceptual understanding of what addition is.

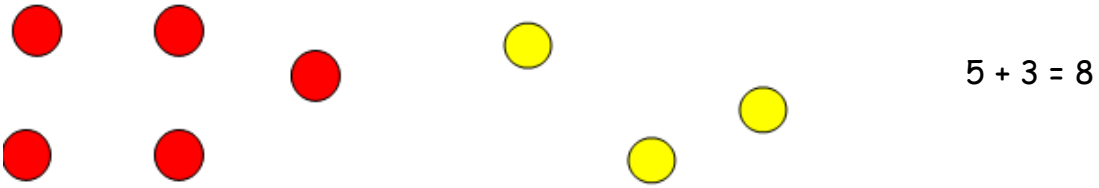
Children will be learning how to count objects, match to the numeral, find and write the correct number next to them.



Children will then learn to combine and count two sets of objects (aggregation).



Next, children will learn to add on to an existing set (augmentation). This means that they will start from 5 and count 3 more to 8.

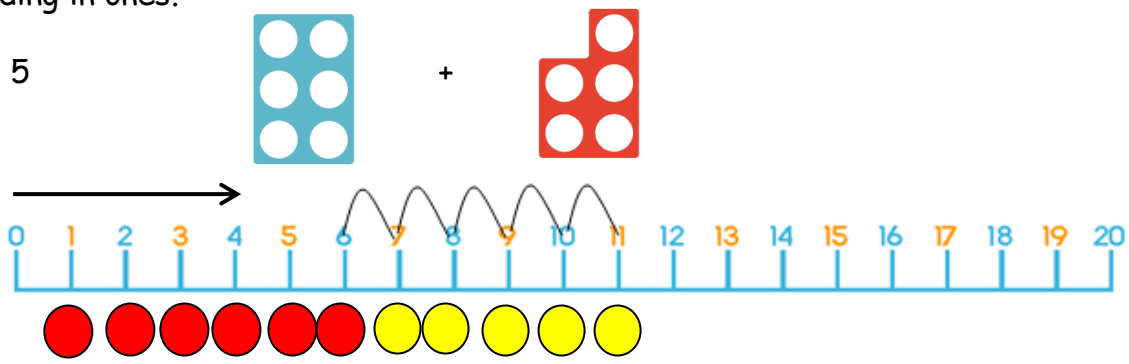


Augmentation is the first step towards a written calculation and children will be using the '+' and '=' symbols.

Stage 2 – Using a numberline alongside the practical resources

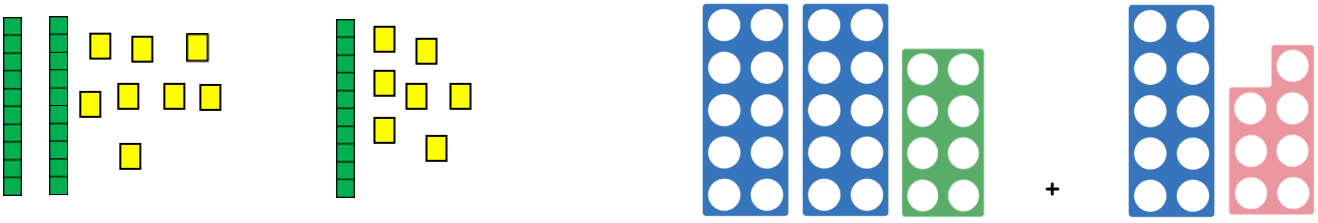
Adding in ones.

6 + 5

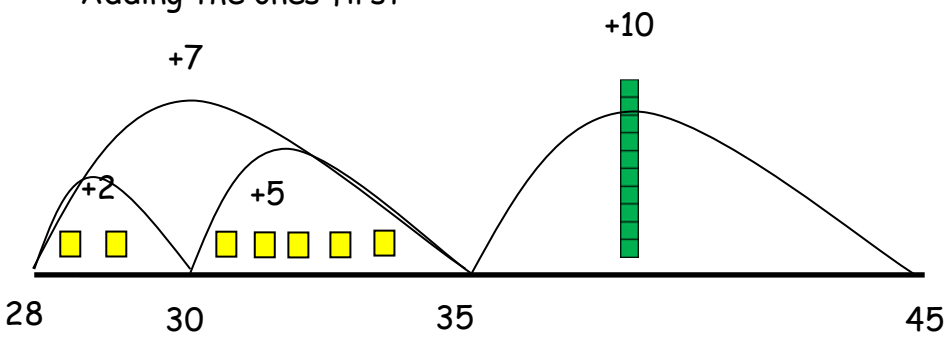


Children are encouraged to use their knowledge of number bonds to help them partition numbers.
As children become more confident with numbers, they will begin to use an empty number line. They will be using base 10 materials or place value counters.

They can now add in bigger jumps.
28 + 17



Adding the ones first



Add the ones then the 10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Other Opportunities at this stage should include:

- **Children also need to opportunities to add more than 2 numbers together.**
- **Ensure that estimation is encouraged and then checked using the inverse operation.**
- **Introducing the children to the inverse operation concept is essential. The should be shown that 12 + 5 = 17 and 5 + 12 = 17 so 17 – 5 = 12 and 17 – 12 = 5**

It is expected that Numicon is used to support the understanding of addition.

Please refer to the Numicon resources for the many ways that this resource can be used to support the understanding.

Stage 3 – Expanded Column Method – using partitioning

The children now need to move to a more refined method. The expanded column method prepares the children well for stage 4, the refined column method.

Once again, practical resources can be used alongside this method to support the understanding (see below).

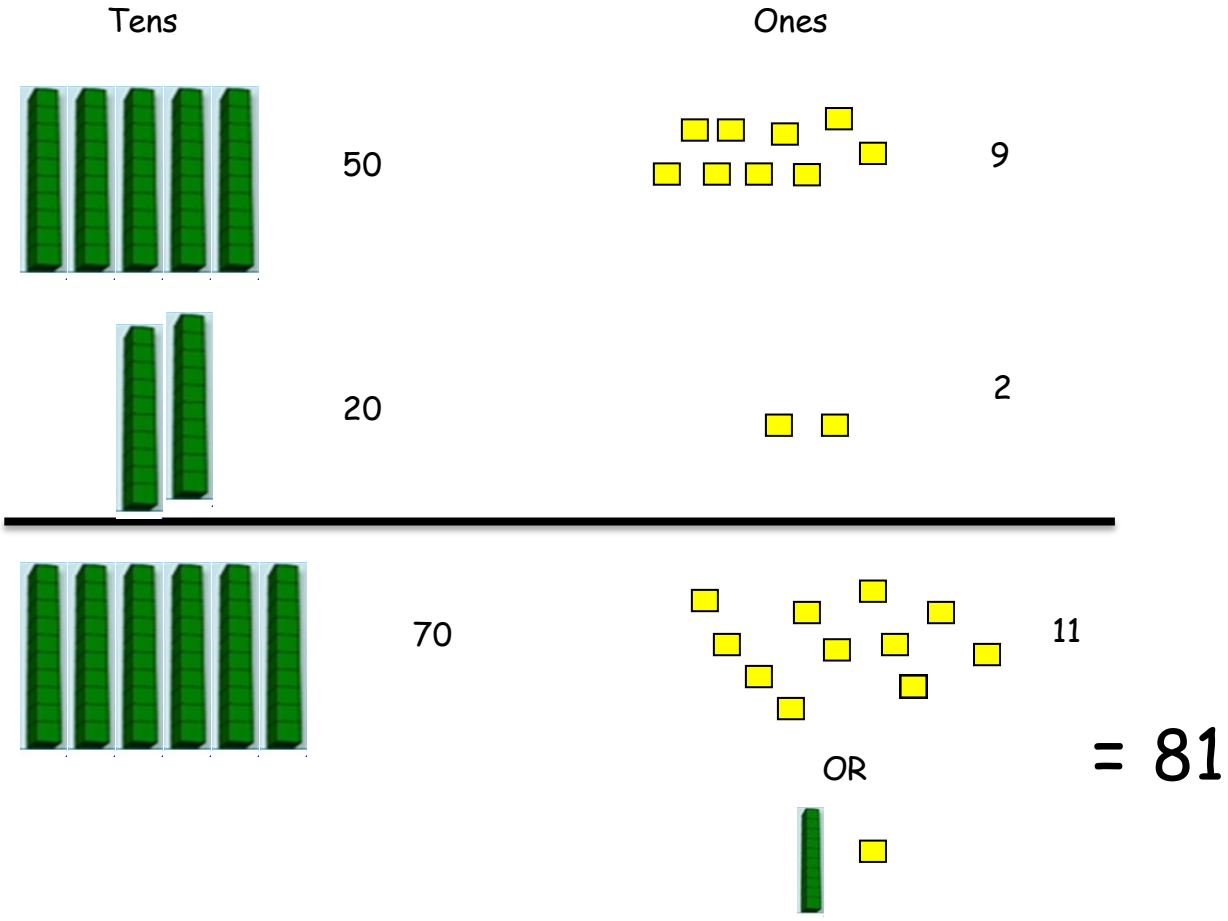
59 + 22 =

	50	**9**
	20	**2**
70	**+ 11**	**= 81**

- **Ensure that estimation and checking using the inverse operation are a part of everyday practise.**
- **Adding more than 2 two digit numbers is also expected at this stage.**

The place value counters and base 10 materials help the children progress towards a more familiar written algorithm.

59 + 22



Move into HTO + TO

387 + 249

300	**80**	**7**
200	**40**	**9**

500+ 120 + 16 = 636

then money (use coins alongside this method initially)

£2.83 + £1.78

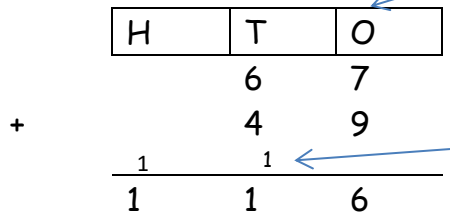
£2	**80p**	**3p**	
£1	**70p**	**8p**	
£3	**+ 150p + 11p**		**= £4.61**
	or £1.50		

Stage 4 – the refined column method

This method needs to be introduced alongside the expanded method (stage 3) to ensure that the children make the link between the two methods.

Avoid phrases, such as, 'carrying'. Explain to the children that the 'ten number goes in the tens' column to be added with the tens' or 'the hundred number goes in the hundreds' column to be added with the hundreds'.

67 + 49



The place value columns can be omitted when secure.

The 'ten' goes in the tens' column to be added with the other tens.

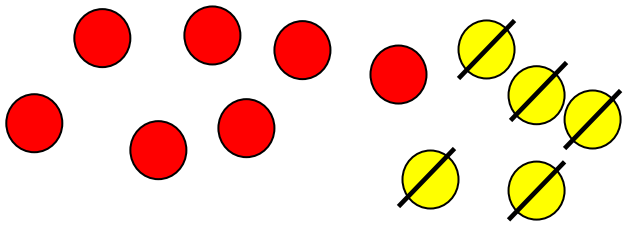
The numbers will sit on the line (not under)

The majority of children should be using the refined column method by the end of Y6.

Subtraction

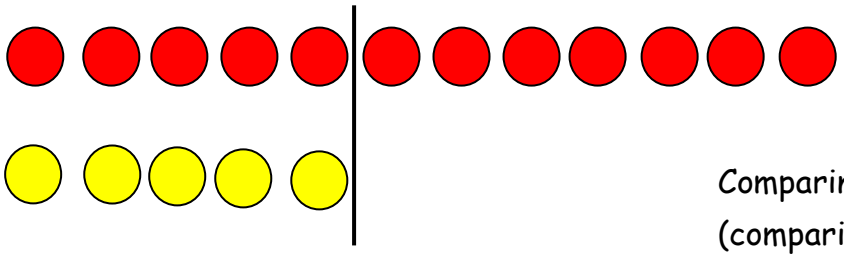
Stage 1 – Various practical activities to ensure a conceptual understanding of what suntraction is.

Children will be learning how to subtract using different objects or toys to support them.



Removing items from a set (reduction or take-away)

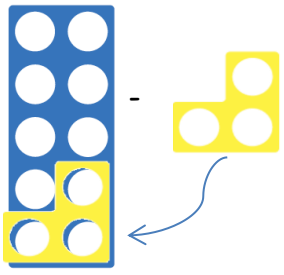
(12 – 5 = 7) The notation will come later after practical activities & language is secure.



Comparing a set of objects (comparison or difference) – This helps children understanding the inverse and commutative nature of Mathematics.

It is expected that Numicon is used to support the understanding of subtraction.

Please refer to the Numicon resources for the many ways that this resource can be used to support the understanding.

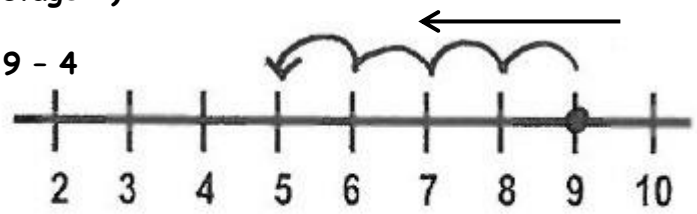


Lay the Numicon on top to show 10-3 = 7
This introduces 'finding the difference'

Stage 2 – counting back on a numberline

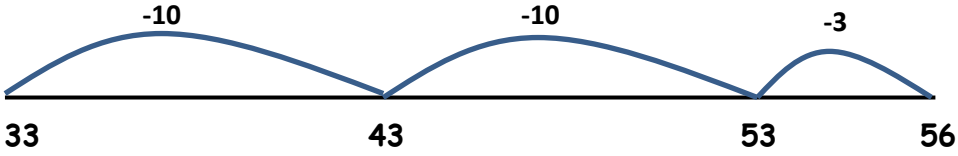
This method would be modelled to the children alongside the practical resources (as shown in stage 1)

9 – 4



56 – 23 =

Subtract the 'ones' first



Counting back can also be modelled using a number square (grid). Children should know that moving to the left on a grid is -1 and moving up is -10.

56 – 23 = 33

Subtract the ones first

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55		57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

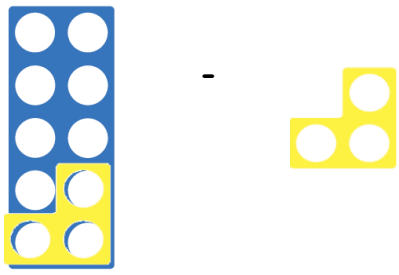
Introducing the children to the inverse operation concept is essential and should be modelled together. This should be shown as

12 + 5 = 17 and 5 + 12 = 17 so 17 – 5 = 12 and 17 – 12 = 5

Stage 3 – Counting on using a numberline – finding the difference.

Once the children are secure with what subtraction is (counting back) and can show this using practical equipment, move the children onto the 'counting on' method – finding the difference.

Revisit using the Numicon to secure the conceptual understanding.

10 – 3 =  -

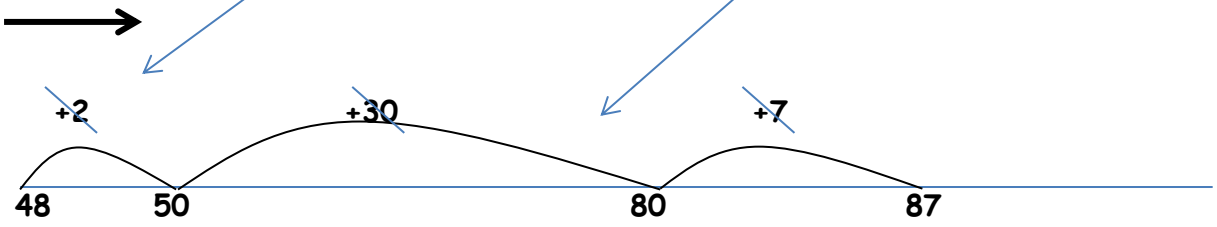
Encourage 'counting on' from 3 to 10

So 3 + 7 more equals 10.

87 – 48 =

Always go to the next 10

The 10 below the number – this can be done in 10s initially.



Re-combining

30 + 7 + 2

37 + 2 = 39

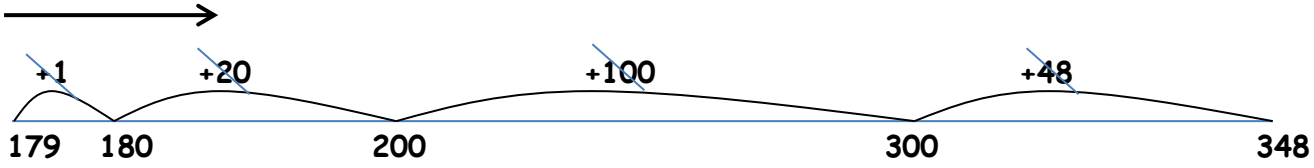
Encourage the children to use the inverse to check their answer e.g. 39 + 48 = 87

The 'jumps' need to be added up below the numberline. Encourage the children to add the largest number first and cross them out as they go along to help to avoid mistakes.

Provide opportunities to subtract HTO – TO (e.g. 243 – 86)

When ready for larger numbers…

348 - 179



100 + 48 + 20 + 1

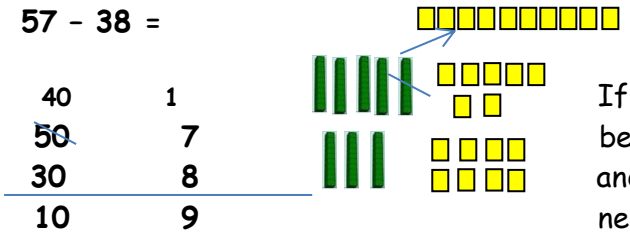
148 + 21 = 169

This is an example of a jotting to support a mental calculation so can be shown in different ways.

Stage 4 – The Expanded Column Method – using partitioning

The language and explanation used when modelling this method is important and needs to be consistent.

57 – 38 =



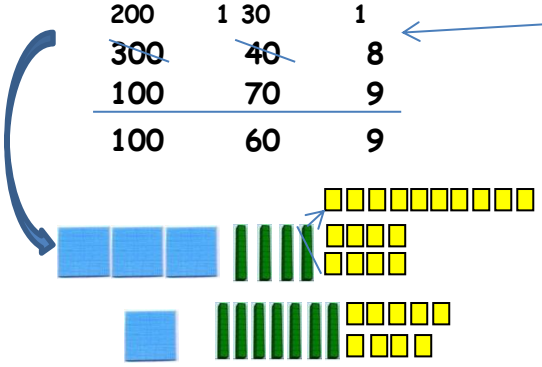
40	1
~~50~~	7
30	8
10	9

Use this explanation

If you have 7 'ones' you do not have enough 'ones' to be able to subtract (take away) 8 – (in this context and without going into negative numbers) - so a ten needs to be **exchanged**.

348 – 179

200	1 30	1
~~300~~	~~40~~	8
100	70	9
100	60	9



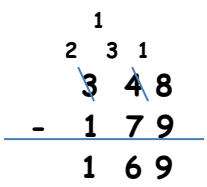
If you have 8 'ones' you do not have enough 'ones' to be able to subtract (take away) 9 – (in this context and without going into negative numbers) - so a ten needs to be **exchanged**.

Please do not use other phrases, such as, 'borrowing', 'knocking on the door' etc.

This method should initially be modelled alongside the Base 10, Numicon or other equipment. Physically show the children the exchanging process.

Stage 5 – The Refined Column Method

348 – 179 =



	1		
	2	3	1
	~~3~~	~~4~~	8
-	1	7	9
	1	6	9

The term 'exchange' must be used throughout this method.

'If you have 8 ones you do not have enough ones to be able to subtract (take away) 9 – (in this context and without going into negative numbers) - so a ten needs to be **exchanged for 10 ones**.

Please do not use other phrases, such as, 'borrowing', 'knocking on the door' etc.

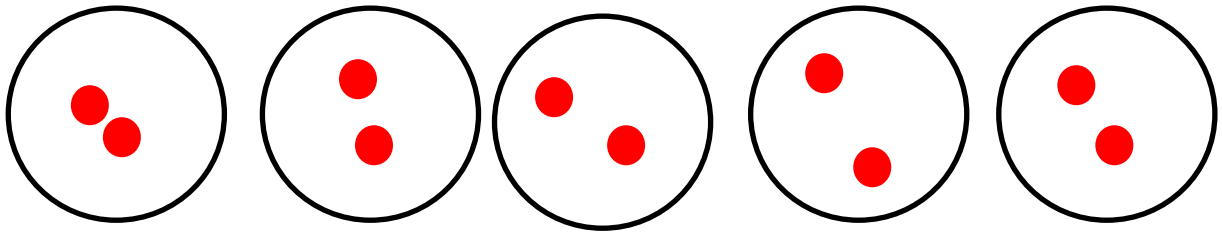
Multiplication

The majority of children should be using the grid method securely before leaving Y6. Some children, the HA, can progress to the short multiplication method.

4 x3 should be explained as 4 three times (4+4+4). The starting number is the number you start with and the x? tells you what to do with it. So, the calculation becomes 4, multiplied by 3.

Stage 1 – Counting in 2s, 5s, 10s using practical resources.

2 x5



2 4 6 8 10

Use socks, cubes and a range of other resources to model the concept of multiplication.

2 4 6 8

2 + 2 + 2 + 2

2 x4

3 x4

3 6 9 12

3 + 3 + 3 + 3

Use the language '4 groups of 3' and '4 lots of 3' with the children but emphasise 'groups of' as this will then support the next stage.

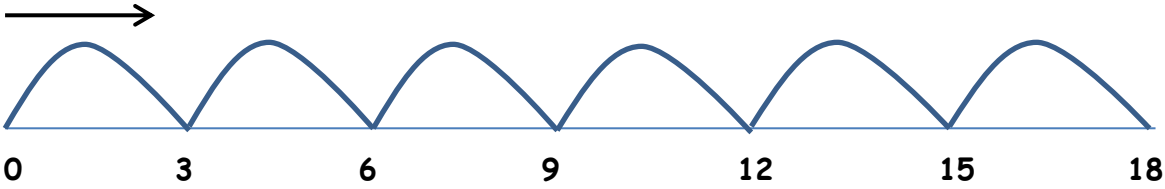
Stage 2 – Repeated Addition

Use the Numicon to model this concept.

3 x6

3 + 3 + 3 + 3 + 3 + 3

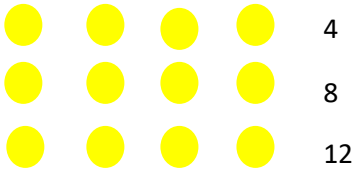
This can then be linked to a numberline. Show these images together.



Also, use counters alongside the numberline to show this in various ways.

Arrays

Children will become familiar with using arrays to model x tables.

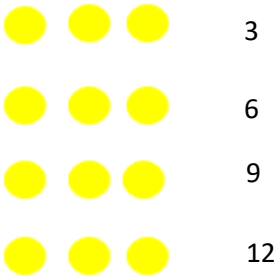


4

8

12

4 x3



3

6

9

12

3 x4

Stage 3 – Partitioning

When practising written methods, avoid multiplying by 2, 4 or 5. The children need to consider more effective mental methods for these calculations (e.g. x4 is double and double again).

Multiplying by a single digit

23 x6 =
3 x6 = 18
20 x6 = 120

120 + 18 = 138

Grid Method

X	30	5
7	30x7 3x7 = 21 210	7x5 7x5 = 35 35

Show the calculations in each section.

When the grid is complete, add up the answers outside of the grid.

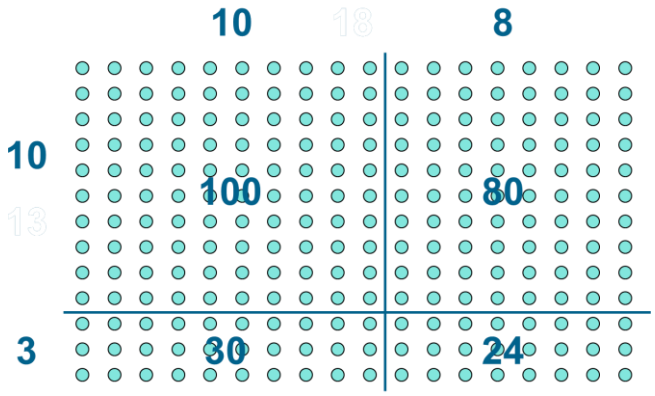
210 + 35

The jottings inside the grid can be dropped when secure.

Show the children what the grid represents in a visual way to ensure understanding.

18 x13 =

X	10	8	
10			
3			



35 x26

X	30	5
20	20x30 2x3 = 6 600	20x5 2x5 = 10 100
6	30x6 3x6 = 18 180	6x5 30

600 + 100 + 180 + 30 (The children may wish to show this in a column method format.)

The HA children can progress to the most refined column method.

Stage 4

This stage will only be used when the children are showing a very secure understanding of stage 3. Most children will leave Y6 using the grid method securely.

H	T	O		
	3	**5**	x	
	2	**6**		
	3	0		(5 x6)
1	8	0		(30 x6)
1	0	0		(5 x20)
6	0	0		(30 x20)
9	1	0		

→ **Show the grid method alongside** ↓

×	30	5
20	20x30 2x3 = 6 600	20x5 2x5 = 10 100
6	30x6 3x6 = 18 180	6x5 30

H	T	O		
	3	**5**	x	
	2	**6**		
	3			
2	1	0		+
1				
7	0	0		
9	1	0		

→

Show this initially…

35 x6 = 210

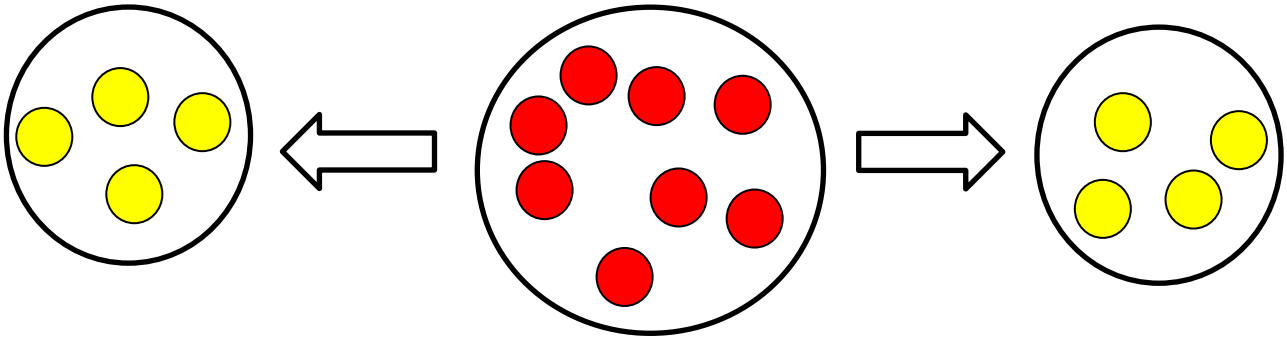
35 x20 = 700

Division

The majority of children should be using the 'chunking' method securely by the end of Y6.

Stage 1 – Introducing the concept of sharing through the use of a range of practical resources and discussion. Language is important.

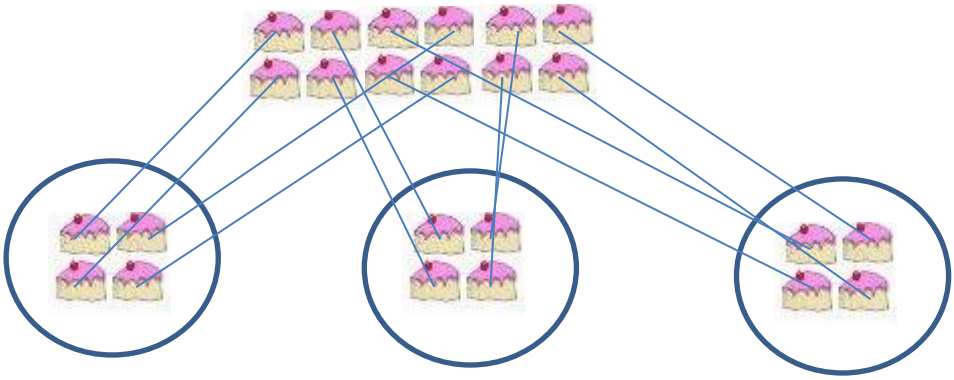
Children will be learning how to share, using different objects or toys to support them.



Share between 2, then 3, etc.

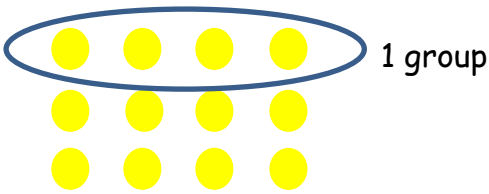
This experience will be brought into school from children's own experiences.

Share 12 cakes shared between 3 people (12÷3=)



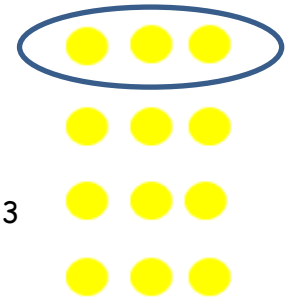
Stage 2 - Grouping

Sharing becomes inefficient as children learn bigger numbers. Grouping should then be modelled.



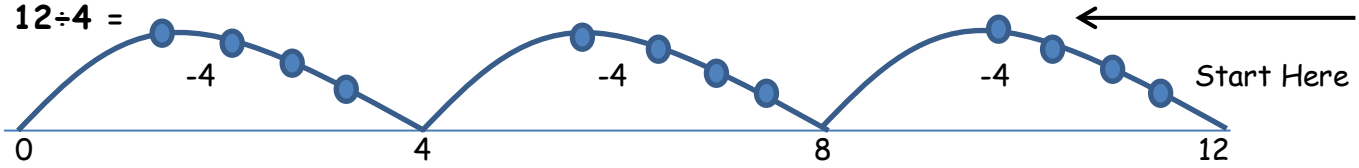
1 group

3 groups of 4
4 x3



4 groups of 3
3 x4
12÷3

12÷4 =



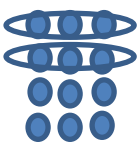
Stage 3 – Counting back on a numberline – repeated subtraction

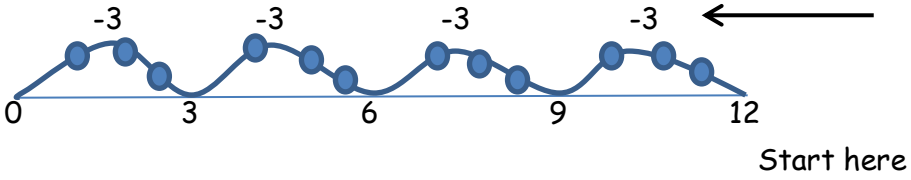
When practising written methods, avoid dividing by 2, 4 or 5. The children need to consider more effective mental methods for these calculations (e.g. ÷4 is half and half again).

Ensure that the children have a clear understanding of what division is – make the link between sharing and grouping e.g.

'Division is when objects are shared. Sharing is not an effective method when dealing with larger numbers so another approach is to group the objects. Division is subtracting groups of objects.

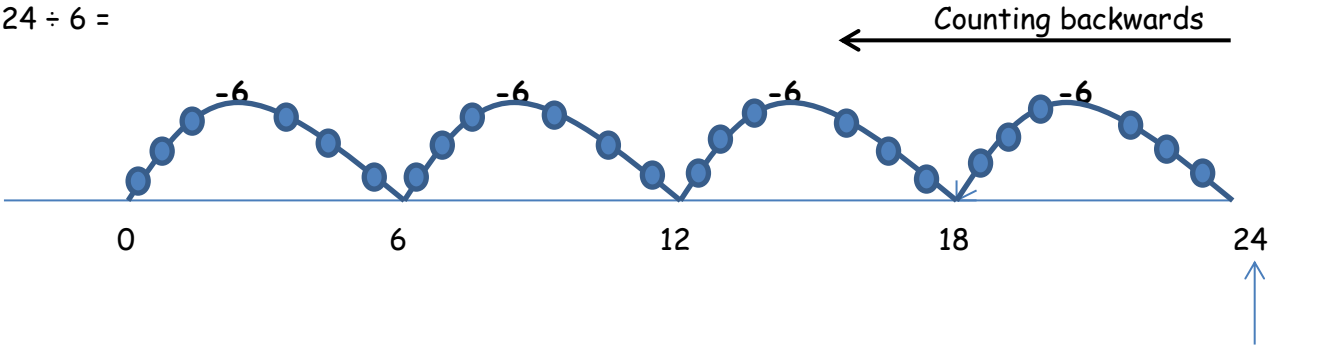
12÷3 = starting with 12 objects and then subtracting (taking away) groups of 3 to find out how many groups of 3 there are in 12. This is repeated subtraction.'





Model this method alongside practical equipment.

24 ÷ 6 =



Show the children 24 objects. Subtract groups of 6 and place along the numberline. **The answer is 4 groups of 6 equal 24 because 6 x4 =24.**

Using Numicon to support understanding.

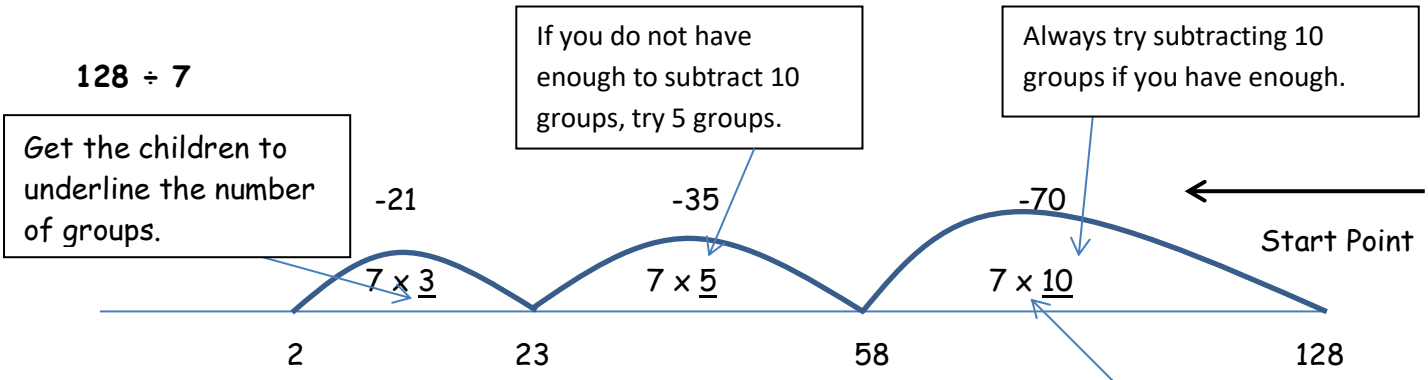
One way it can be used – lay out the number being divided e.g. 24 ÷ 6
Place as many of the '6' Numicon shapes on top as will fit.
This is also good to show the concept on 'remainders'.

Provide opportunities for the children to experience remainders using this method, e.g. 32÷7= 4r 4. Get the children to use the inverse operation to check the answer (7x4=28 + 4).

Stage 4

When secure, show the children how to subtract multiple groups. Explain how this strategy is required when the numbers are larger to ensure greater efficiency.

128 ÷ 7



After completing the subtraction of the groups, count the number of groups – if these are underlined it makes it clearer.

Ensure that the number dividing by is written first (it is 7 multiplied by 10 or subtracting 10 groups of 7).

Stage 4 – The 'Chunking' Method

This method requires the children to be secure with column subtraction (which should be secure before the children enter Y6)

Initially, show the children this method alongside the number line – stage 2.

196 ÷ 6 =

6)196	
− 60	6 × 10
136	
− 60	6 × 10
76	
− 60	6 × 10
16	
− 12	6 × 2
4	32
Answer:	32 R 4

The number you are dividing by (6 in this case) goes first. It is 6 multiplied by 10.

OR $32\ \frac{4}{6}$

The children should be encouraged to estimate before carrying out any calculation.

The inverse operation should then be used to check the answer.
The HA may move on to the short division method and may also show remainders as a fraction e.g. $\frac{4}{6}$

Children will become more efficient with practise & modelling.

196 ÷ 6

6 | 196		
-180	6 x30	
16		
- 12	6 x2	32 r4
4		

Stage 5 – Short Division

This stage will only be used when the children are showing a very secure understanding of stage 4. Most children will leave Y6 using the 'Chunking Method' securely.

$$\begin{array}{r} 032\ r\,4 \\ 6\ \overline{)\,19^{1}6} \end{array}$$